Comment on "Magnon wave forms in the presence of a soliton in two–dimensional antiferromagnets with a staggered field"

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Very recently Fonseca and Pires [Phys. Rev. B **73**, 012403 (2006)] have studied the soliton–magnon scattering for the isotropic antiferromagnet and calculated "exact" phase shifts, which were compared with the ones obtained by the Born approximation. In this Comment we correct both the soliton and magnon solutions and point out the way how to study correctly the scattering problem.

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The soliton–magnon interaction in 2D magnets is a subject of an intensive studying more than 10 years. The problem of a magnon scattering by the Belavin–Polyakov soliton in isotropic magnets, in particular, antiferromagnets was solves by Ivanov et al.[1]. In a recent paper Fonseca and Pires[2] come back to the soliton–magnon scattering problem in an isotropic antiferromagnet. The reason is that authors propose to consider a new type of soliton in the isotropic antiferromagnet, which is characterized by its internal precession. In the paper[2] authors consider also the influence of a staggered magnetic field.

In order to describe the soliton structure, the angular parametrization of the sublattice magnetization vector is involved, $\boldsymbol{n} = \{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$. The soliton structure is described by the singular distribution of the ϕ–field, $\phi_s = q\varphi - \Omega t$ and the regular one for the θ–field:

$$\frac{\mathrm{d}^2\theta_s}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\theta_s}{\mathrm{d}r} + \left(k_0^2 - \frac{q^2}{r^2}\right)\sin\theta_s\cos\theta_s = h\sin\theta_s, \quad (1a)$$

$$\theta_s(0) = \theta_s(\infty) = 0. \quad (1b)$$

Here r and φ are the polar coordinates in the XY plane, $k_0 = \Omega/c$. Multiplying Eq. (1a) by $r^2\mathrm{d}\theta_s/\mathrm{d}r$ and integrating over all r with account of boundary conditions (1b), one can easily obtain the identity[3]

$$k_0^2\int_0^\infty \sin^2\theta_s(r) r\mathrm{d}r = h\int_0^\infty \left[1-\cos\theta_s(r)\right] r\mathrm{d}r. \quad (2)$$

Note that the identity (2) can be satisfied only for $\theta_s(r) \equiv 0$ in the case of $h \leq 0$. However namely this case, $h \leq 0$, corresponds to the ground state $\theta_0 = 0$: it minimizes the energy functional (3) of the paper by Fonseca and Pires[2]. The differential problem (1) also has only the trivial solution for $h > 0$, this results from the phase plane analysis.

Thus one can conclude that the differential problem (1) has *only* the trivial solution $\theta_s(r) = 0$; hence it has no sense to consider some nontrivial distribution in the ϕ–field, because the soliton does not exist.

Besides the localized soliton solution Fonseca and Pires[2] mention also nolocalized vortex–like solutions. In principle, it is possible to discuss such solutions when $h > 0$, because the ground state becomes $\theta_0 = \arccos H$, see Eqs. (9) of the paper by Fonseca and Pires[2]. However the energy of such solution does not have a logarithmic divergence like (12) and (13): the correct form is mainly determined by the term

$$\frac{J}{2c^2}\int \mathrm{d}^2x \sin^2\theta\left(\frac{\partial\phi}{\partial t}\right)^2 \propto R^2$$

and diverges as the system area, so the precessional vortex solution also is not preferable.

For the soliton–magnon scattering problem authors come back to the localized solution. In order to consider magnons in a presence of the soliton, the following ansatz is involved: $\theta(\boldsymbol{r},t) = \theta_s(\boldsymbol{r}) + \eta(\boldsymbol{r},t)$. Here authors neglect the out–of–plane soliton structure, $\theta_s = 0$, which corresponds to our conclusion about the absence of the soliton solution. Thus, magnons are considered on the following background:

$$\theta_s(r) = 0, \qquad \phi_s(\varphi,t) = q\varphi - \Omega t. \quad (3)$$

After linearization of Eq. (4) of the paper authors obtain Eq. (14), which leads to the scattering picture.

The reasonable question is how the "soliton" (3), which is simply a ground state, can scatter magnons? Authors chose the plane–wave solution of the form $\eta(\boldsymbol{r},t) = \exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t)$, which is not a correct mathematical object, because the real scalar η can not be identified with the complex exponent. The correct form is the real quantity $\eta(\boldsymbol{r},t) = A\cos(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)$, $\phi = \text{const}$, which describes the linearly polarized spin wave. However this linearly polarized wave is not compatible with the solution (3): after the substitution into Eq.(5) of Ref. 2, one can obtain the following equation

$$\frac{q}{r^2}\frac{\partial\eta}{\partial\varphi} = -\frac{\Omega}{c^2}\frac{\partial\eta}{\partial t},$$

which can not be solved together with Eq. (14), but authors do not take it into account. This cause also the wrong dispersion law (15a).

The correct way is to consider the circular polarized spin wave of the form $\theta = \text{const} \ll 1$, $\phi = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t$, which has the same symmetry as a "soliton" solution (3). After that the magnons on the soliton background are described by the linear corrections both to θ and ϕ components and the magnon solution on the background (3) has

the form similar to Eq. (17) for both corrections. However instead of nonanalytic dependence $\mu = \sqrt{n^2 + q^2}$, the correct index of the Bessel function has a form $\mu = |n+q|$, see e.g. Ref. 1. In the case of the solution (3) the role of the q–term is the redefinition of the azimuthal quantum numbers n. Therefore the"exact solution" (17) of the scattering problem should be reexamined.

We want to stress also that the Born approximation is *not* adequate for the soliton–magnon scattering problem, see a discussion in Ref. 4.

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